B-field Determination from Magnetoacoustic Oscillations in kHz QPO Neutron Star Binaries: Theory and Observations

L. G. Titarchuk

George Mason University/CEOSR, Fairfax VA

lev@xip.nrl.navy.mil

C. F. Bradshaw

School of Computational Sciences, George Mason University, Fairfax VA

cbradshaw@tstag.com

K.S. Wood

Naval Research Laboratory, Washington DC

wood@ssd0.nrl.navy.mil

ABSTRACT

We present a method for determining the B-field around neutron stars based on observed kHz and viscous QPO frequencies used in combination with the best-fit optical depth and temperature of a Comptonization model. In the framework of the transition layer QPO model, we analyze magnetoacoustic wave formation in the layer between a neutron star surface and the inner edge of a Keplerian disk. We derive formulas for the magnetoacoustic wave frequencies for different regimes of radial transition layer oscillations. We demonstrate that our model can use the QPO as a new kind of probe to determine the magnetic field strengths for 4U 1728-42, GX 340+0, and Sco X-1 in the zone where the QPOs occur. Observations indicate that the dependence of the viscous frequency on the Keplerian frequency is closely related to the inferred dependence of the magnetoacoustic wave frequency on the Keplerian frequency for a dipole magnetic field. The magnetoacoustic wave dependence is based on a single parameter, the magnetic moment of the star as estimated from the field strength in the transition layer. The best-fit magnetic moment parameter is about $(0.5-1)\times10^{25}$ G cm^3 for all studied sources. From observational data, the magnetic fields within distances less 20 km from neutron star for all three sources are strongly constrained to be dipole fields with the strengths 10^{7-8} G on the neutron star surface.

Subject headings: accretion disks — stars: neutron — stars: individual (4U 1728-34, Scorpius X-1; GX 340+0)— stars: magnetic fields— X-rays: bursts

1. Introduction

Strong magnetic fields of neutron stars, typically of order $\sim 10^{12}$ G (Bhattacharya, & Srinivasan 1995) are considered crucial to the radio pulsar activity but equilibrium arguments concerning NS magnetic field pressure and ram pressure balance lead to magnetic field strength estimates for heavily accreting NS sources (for example X-ray bursters) of $\lesssim 10^9$ G. In these sources no persistent pulsations have been found despite careful searches (Wood et al. 1991; Vaughan et al. 1994). Different scenarios of magnetic field evolution lead to different estimates of magnetic field strength and decay. Thus, it is desirable to find a direct way of measuring or estimating the neutron star's magnetic field in these binary systems. Any model used to make such estimates should have minimum assumptions about observed data and model relationships to produce accurate results. *Our model, described herein, has only one parameter that can be adjusted for agreement, the magnetic moment. All other parameters are strongly constrained by observations.* It develops a new probe of the magnetic field strength and geometry, based on radial magnetoacoustic oscillations.

Titarchuk, Lapidus & Muslimov (1998), hereafter TLM98, considered the possibility of dynamical adjustments of a Keplerian disk to the innermost sub-Keplerian boundary conditions to explain most observed QPOs in bright low mass X-ray binaries (LMXBs). They concluded that an isothermal transition sub-Keplerian layer between the NS surface and its last Keplerian orbit forms as a result of this adjustment. The TLM98 model is general treatment applicable to both NS and black hole (BH) systems. The primary problem in both NS and BH systems is understanding how the flow changes from pure Keplerian to sub-Keplerian as a radius decreases to small values. The TLM98 authors suggested that the discontinuities and abrupt transitions in their solution result from derivatives of quantities such as angular velocities (weak shocks). Subsequently, Titarchuk & Osherovich (1999 hereafter, TO99) identified the transition layer (TL) model's corresponding TL viscous frequency ν_V and diffusion or break frequency ν_b for 4U 1728-34, and predicted values for ν_b related to diffusion in the boundary layer. TLM98 argued that a shock should occur where the Keplerian disk adjusts to sub-Keplerian flow. They interpreted the low frequencies as acoustic oscillations and expected them to provide reliable estimates of radial velocities that would be very close to the acoustic velocities.

Sco X-1's derived acoustic velocities (Titarchuk et al. 2001 hereafter TBGF) translated to a TL plasma temperature, assuming only thermal motion, of about 19 keV. However

photon spectral fitting by TBGF yielded a TL temperature of 2.8 keV. TBGF attributed this discrepancy to a dominant contribution to the plasma matter energy density from the magnetic field. *The viscous QPOs provide information identifiable with a magnetoacoustic (MA) oscillation effect that can be modeled.*

In this *Letter*, we develop a general formalism for MA radial oscillations in the TL. In §2, we describe the data sources used to test our MA oscillation model. The formulation of the problem and derivation of MA frequencies are described in §3. We show the results of the MA oscillation interpretation for the TO99 QPO viscous frequencies and the best-fit normalization parameters of magnetic field strengths for a number of QPO sources are present in §4. Our summary and conclusions follow in §5.

2. Observations

The model can only be applied to QPOs in which the full TLM has been worked out and in which the QPOs identified with oscillations of a bounded medium have been seen. All observations[1] referenced herein were made with the Rossi X-ray Timing Explorer's (RXTE). Sco X-1 data were obtained during August 3 and 22, 1997, February 27 and 28, 1998, and June 10-13, 1999. Data were extracted from research archives for GX 340+0 and 4U 1728-34 for observations on November 14, 1998 and February 28, 1999, respectively. In addition, we use the results from observations of 4U 1728-32 between February 15-March 1, 1996 by Ford & van der Klis (1998)

3. Magnetoacoustic Oscillations in the Transition Layer

In the references cited above, the treatment of a magnetic field in a fluid disk has specified neither the multipole order that characterizes the field nor the strength of the field.

We derive the frequency of the QPO associated with MA oscillations and the correlation of the MA frequency with the Keplerian frequency $\nu_{\rm K}$. The MA frequency is derived as the eigenfrequency of the boundary-value problem resulting from a MHD treatment of the interaction of the disk with the magnetic field. The problem is solved for two limiting boundary conditions encompassing realistic possibilities. The solution yields a velocity identified as a mixture of the sound speed and the Alfvén velocity, becoming either at the appropriate lim-

[1]This research has made use of data obtained through the High Energy Astrophysics Science Archive Research Center Online Service, provided by the NASA/Goddard Space Flight Center.

its. Our treatment does not specify how the eigenfrequency is excited or damped. However, it makes clear that the QPO is a readily stimulated resonant frequency.

In order to derive the MA oscillation frequencies in the isothermal transition layer, we consider layer perturbations such that the density $\rho = \rho_0 + \rho_1$, the velocity $\mathbf{v} = \mathbf{v_0} + \mathbf{v_1}$ and $\mathbf{B} = \mathbf{B_0} + \mathbf{B_1}$. By combining the continuity equation, equation of motion, and an ideal gas equation, we obtain an equation for MA oscillations (Alfvén 1942)

$$\frac{\partial^2 \mathbf{v_1}}{\partial t^2} - s^2 \nabla(\nabla \cdot \mathbf{v_1}) + \mathbf{v}_\mathrm{A} \times \nabla \times [\nabla \times (\mathbf{v_1} \times \mathbf{v}_\mathrm{A})] = F(\mathbf{v_0}, \partial \mathbf{v_0}/\partial \mathbf{r}, \mathbf{B_0}, \rho_0) \quad (1)$$

where $s^2 = (\partial P/\partial \rho)$ is the square of the sound velocity (assumed constant with respect to radius) and $\mathbf{v}_\mathrm{A} = \mathbf{B_0}(4\pi\rho_0)^{-1/2}$ is the Alfvén velocity. To determine the eigenfrequencies of MA oscillations in the TL, we set equation (1) to zero and introduce a cylindrical coordinate system in the TL with a radial axis in the disk plane, z-axis perpendicular to the plane, and azimuthal axis along the circle in the disk plane [see Fig. 1 in Titarchuk, Osherovich & Kuznetsov (1999), hereafter TOK, for the transition layer geometry].

The magnetic field acts along a direction in space that is the same throughout the TL, intersecting the disk everywhere at the same angle δ. Practical ranges for δ are from a few to ~ 15 degrees (Titarchuk & Osherovich 2001). The model used for all three sources (see §4) assumes that $\delta = 0$ exactly, which reinstates axial symmetry and makes the problem mathematically tractable. The simplified problem is spatially one dimensional in the radial direction, and oscillations represented by the eigenfunctions are radial fluid movements.

We consider the radial oscillations in the TL for which we should assume that $\mathbf{B_0} = B_0 \mathbf{e_z}$, i.e., the mean magnetic field strength B_0 is perpendicular to the disk and $\mathbf{v_1} = v_1 \mathbf{e_r}$ is along the radial direction in the disk. We wish to explore how solutions for flow in the TL are affected by boundary conditions and B field geometry, for acoustic and magnetic oscillations. Under these assumptions, equation (1) can be rewritten as:

$$\frac{\partial^2 v_1}{\partial t^2} - s^2 \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_1}{\partial r}\right) - v_\mathrm{A} \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_\mathrm{A} v_1}{\partial r}\right) = 0. \quad (2)$$

Combining this equation with the two boundary conditions of the TL, the neutron star surface $r = R_{in}$ and the outer boundary, $r = R_{out}$, defined by the Keplerian QPO frequency, allows us to analyze the eigenfrequencies of the MA oscillations in the TL under both the stiff $v_1(R_{in}) = v_1(R_{out}) = 0$ and the free $\partial v_1/\partial r(R_{in}) = \partial v_1/\partial r(R_{out}) = 0$ boundary conditions.

In the two extreme cases, either $v_\mathrm{A} \ll s$ or $v_\mathrm{A} \gg s$ equation (2) can be examined analytically under the appropriate boundary conditions. Introducing a new function $y = r v_1$ in the case of $v_\mathrm{A} \ll s$ and, $y = r v_\mathrm{A} v_1$, in the case of $v_\mathrm{A} \gg s$, we can write equation (2) in

the form

$$\frac{\partial^2 y}{\partial t^2} = 4Ax^{(2-\alpha)/2}\frac{\partial^2 y}{\partial x^2} \tag{3}$$

where $x = r^2$ is a new variable, $\alpha = 0$ and $A = A_s = s^2$ for the pure acoustic case, and $\alpha \geq 6$ and $A = A_m = B_*^2 r_*^\alpha / 4\pi\rho$ for the magnetic case. The index α is related to the multipole magnetic field, $v_{\mathrm{A}}^2 = A_m/r^\alpha$ ($\alpha = 6, 8, 10$ are for the dipole, quadrupole, and octopole, respectively). B_* is the magnetic field strength at a radius $R_{in} < r_* < R_{out}$, and ρ is the TL mean density.

First, we need to find a solution of the eigenvalue problem for equation (3) under the stiff boundary conditions by obtaining a particular nontrivial solution using the separation variables method, such that

$$y(x,t) = X_\lambda(x)T_\lambda(t) \tag{4}$$

where $X_\lambda(x)$ is an eigenfunction satisfying

$$X'' + \frac{x^{(\alpha-2)/2}}{4A}\lambda^2 X = 0 \tag{5}$$

and boundary conditions $X(x_{in}) = X(x_{out}) = 0$. The solution of these equations can be expressed through the Bessel Z functions

$$X_\lambda = x^{1/2} Z_{2/(\alpha+2)}\left[\frac{2}{(\alpha+2)}\frac{\lambda}{A^{1/2}}x^{(\alpha+2)/4}\right]. \tag{6}$$

X_λ is a linear superposition of the Bessel $J_{2/(\alpha+2)}$ and $J_{-2/(\alpha+2)}$ functions:

$$X_\lambda = x^{1/2}[J_{2/(\alpha+2)}(z_\lambda) \ + \ CJ_{-2/(\alpha+2)}(z_\lambda)] \tag{7}$$

where

$$z_\lambda = \frac{2}{(\alpha+2)}\frac{\lambda}{A^{1/2}}x^{(\alpha+2)/4}. \tag{8}$$

Wood et al. (2001) demonstrated that an asymptotic form of the Bessel function for $z_\lambda \gtrsim 1$ can be used to find all eigenvalues including the first one:

$$J_{2/(\alpha+2)}(z_\lambda) = \left(\frac{2}{\pi z_\lambda}\right)^{1/2} \cos[z_\lambda - (\alpha+6)\pi/4(\alpha+2)] \tag{9}$$

and

$$J_{-2/(\alpha+2)}(z_\lambda) = \left(\frac{2}{\pi z_\lambda}\right)^{1/2} \cos[z_\lambda - (\alpha-2)\pi/4(\alpha+2)]. \tag{10}$$

Using these asymptotic forms, the boundary conditions, and equations (7 & 8), the eigenvalues are determined by

$$\cos(z_{out} - \varphi_1)\cos(z_{in} - \varphi_2) - \cos(z_{out} - \varphi_2)\cos(z_{in} - \varphi_1) = 0, \tag{11}$$

where $\varphi_1 = (\alpha - 2)\pi/4(\alpha + 2)$ and $\varphi_2 = (\alpha + 6)\pi/4(\alpha + 2)$. Equation (11) can be reduced to $\sin(z_{out} - z_{in}) = 0$ or $\beta = z_{out} - z_{in} = n\pi; \quad n = 1, 2, 3...$, with a solution

$$\lambda_n = \frac{\alpha + 2}{2} \frac{n\pi A^{1/2}}{r_{out}^{(\alpha+2)/2} - r_{in}^{(\alpha+2)/2}}. \tag{12}$$

The time dependence, $T(t)$ is found from the equation $T'' + \lambda_n^2 T = 0$. Thus, $T(t) = \sin(\lambda_n t + \phi_0) = \sin(2\pi\nu_n t + \phi_0)$ where ϕ_0 is an initial phase. For the acoustic case ($A = A_s = s$ and $\alpha = 0$), the main eigenvalue $\lambda_1 = \pi/(r_{out} - r_{in}) = \pi/L$ and $\nu_s = \nu_1 = \lambda_1/2\pi = s/2L$, where L is the TL radial size.

For the magnetic case ($\alpha \geq 6$)

$$\nu_M = \nu_1 = \frac{(\alpha + 2)B_*}{4(4\pi\rho)^{1/2}[r_{out} - r_{in}(r_{in}/r_{out})^{\alpha/2}]} \left(\frac{r_*}{r_{out}}\right)^{\alpha/2} = \frac{(\alpha + 2)v_{\rm A}(r_{out})}{4[r_{out} - r_{in}(r_{in}/r_{out})^{\alpha/2}]}. \tag{13}$$

For the free boundaries, the boundary conditions for y are

$$\frac{\partial y}{\partial x} + \frac{(\alpha - 2)}{4}\frac{y}{x} = 0 \quad \text{at} \quad x = x_{in} \quad \text{and} \quad x = x_{out}, \tag{14}$$

from which, the transcendental equation for determining the eigenvalue is

$$\tan\beta = -\frac{2(\alpha - 2)\beta}{(\alpha + 2)\{\eta\beta^2/(\eta - 1)^2 + [(\alpha - 2)/(\alpha + 2)]^2\}}, \tag{15}$$

where $\beta = z_{out} - z_{in}$ and $\eta = z_{out}/z_{in} = (r_{out}/r_{in})^{(\alpha+2)/2}$.

Analysis of equation (15) shows that for the values most relevant to observations ($\eta \gtrsim 2$),

$$\beta_M \approx \frac{\pi}{2} + \frac{2}{\pi}[(\pi/2)^2\eta/(\eta - 1)^2 + 1/4] \tag{16}$$

for $\alpha = 6$ and $\beta_s \approx \{1.5/[1 + 1.5\eta/(\eta - 1)^2]\}^{1/2}$ for $\alpha = 0$.

The free-boundary conditions will therefore introduce additional factors appearing in formulas for ν_1 between 0.5 and 1 for the magnetic case and $1/\pi$ for the acoustic case. From this, we conclude that the free-boundary conditions can decrease substantially the magnetic oscillation frequency and must decrease the acoustic frequency by at least a factor of 3 (see Eqs. 17-18 below).

4. Magnetic Field Strength Determination and the Magnetoacoustic Oscillation Frequency

We can construct an approximate formula for the MA frequency ν_{MA} using the asymptotic forms for the two extreme (acoustic and magnetic) cases

$$\nu_{MA} \approx (\nu_s^2 + \nu_M^2)^{1/2} = \{s^2/4(r_{out} - r_{in})^2 + [(\alpha+2)/4]^2 v_{\rm A}^2(r_{out})/[r_{out} - r_{in}(r_{in}/r_{out})^{\alpha/2}]^2\}^{1/2}. \quad (17)$$

The approximate relation in equation (17) is exact if the Alfv́en velocity is assumed to be constant through the TL. The difference, by a factor $(\alpha+2)^2/4$, in coefficients for s^2 and $v_{\rm A}^2$ is due to different behaviors of s and $v_{\rm A}$ as a function of r ($s = const$ and $v_{\rm A} \propto r^{-\alpha/2}$).

Under the free boundary, equation (17) is modified to

$$\nu_{MA} \approx \{(\beta_s/\pi)^2 s^2/4(r_{out} - r_{in})^2 + (\beta_M/\pi)^2[(\alpha+2)/4]^2 v_{\rm A}^2(r_{out})/[r_{out} - r_{in}(r_{in}/r_{out})^{\alpha/2}]^2\}^{1/2} \quad (18)$$

and provides the dependence of ν_{MA} on $\nu_{\rm K}$. This dependence is due to the explicit dependence of r_{out} on the Keplerian frequency $\nu_{\rm K}$:

$$r_{out} = (GM/2\pi\nu_{\rm K}^2)^{1/3}, \quad (19)$$

where G is the gravitational constant and M is the NS mass. To determine s, we define $s = (kT/m_p)^{1/2}$ as the sound speed for the isothermal TL where kT is the best-fit value of the Comptonization temperature. To determine $v_M = B(r_{out})/(4\pi\rho_0)^{1/2}$, we express $\rho = \tau m_p/(\sigma_T L)$ with τ as the best-fit optical depth parameter. These are not adjustable parameters for determining magnetic field strengths.

Thus, we obtain ν_{MA} as a function of only one adjustable parameter, the magnetic-field strength at a TL radius r_*. We can replace this parameter by the multipole moment $B(r_*)r_*^{\alpha/2}$. In the dipole case $\alpha = 6$, the multipole moment is the dipole moment μ (in G cm^3). The procedure for determining the magnetic field strength in the TL may realized if there is an observable dependence between the viscous and Keplerian frequencies.

The spectra of 4U 1728-34, GX 340+0, and Sco X-1 were fitted by the Comptonization model of Sunyaev & Titarchuk (1980) from 8-25 keV, to obtain a plasma temperature (kT) and Thompson optical depth (τ). Figure 1 shows the results from the model and the fitted data for 4U 1728-34, GX 340+0, and Sco X-1 (Ford & van der Klis 1998).

We found that a dipole geometry ($\alpha = 6$) gives a good fit and higher poles are excluded by observed data. It is also evident from equations (17-19) that the MA frequencies are well described by a power law dependence on $\nu_{\rm K}$ when $\nu_M > \nu_s$, i.e., $\nu_{MA} \approx \nu_M \propto \nu_{\rm K}^{(\alpha+2)/3}$. The power law index in the dipole case is $(\alpha+2)/3 = 8/3$. The MA frequency dependence on

$\nu_{\rm K}$ is too steep for the quadrupole and the octopole cases [where $(\alpha+2)/3 = 10/3$ and 4, respectively] to be supported by the data. The fits of our MA model to the data provide strong constraints for the boundary condition types where the stiff conditions are ruled out for 1728-34 and the free conditions for GX 340+0. The fit qualities are practically insensitive for $M = 1.2 \pm 0.2 M_\odot$. The best-fit mass parameter is around 1.2.

The best-fit parameters for 4U 1728-34 (Compton values $kT = 8.6 \pm 2$ keV, $\tau = 5.8 \pm 0.5$; $\chi^2/dof = 19.8/35$) are $B_* = (0.86 \pm 0.03) \times 10^6$ G, and $r_* = 1.69 \times 10^6$ cm, corresponding to a magnetic moment $\mu = 4.2 \times 10^{24}$ G cm^3. We fixed the NS radius, r_{in} to be (as a canonical value) three Schwarszchild radii 3 $r_{\rm S}$. An extrapolation of the magnetic field from r_* towards the NS radius r_{in} gives us $B_{NS} = 4.5 \times 10^6$ G and $B_{NS} = 1.3 \times 10^7$ G for the dipole and octopole fields, respectively.

The best-fit parameters for GX 340+0 (Compton values $kT = 2.87 \pm 0.02$ keV, $\tau = 11.2 \pm 0.2$; $\chi^2/dof = 49.8/35$) are $B_* = (0.5 \pm 0.05) \times 10^6$ G and $r_* = 2.7 \times 10^6$ cm, corresponding to a magnetic moment $\mu = 1 \times 10^{25}$ G cm^3. An extrapolation of the magnetic field from r_* towards the NS radius r_{in} yields $B_{NS} = 8.3 \times 10^6$ G and $B_{NS} = 5.3 \times 10^7$ G for the dipole and octopole fields, respectively.

The best-fit parameters for Sco X-1 (Compton values $kT = 2.87 \pm 0.03$ keV, $\tau = 11.3 \pm 0.3$; $\chi^2/dof = 9.9/33$) are $B_* = (1. \pm 0.05) \times 10^6$ G, and $r_* = 2.12 \times 10^6$ cm, corresponding to a magnetic moment $\mu = 9.5 \times 10^{24}$ G cm^3 in the TL. An extrapolation of the magnetic field from r_* towards the NS radius r_{in} gives us $B_{NS} = 8.1 \times 10^6$ G and $B_{NS} = 3.3 \times 10^7$ G for the dipole and octopole fields, respectively.

These derived magnetic-field strengths are for the TL only and can be smaller than the true magnetic field outside of the layer because of its finite conductivity. The NS magnetic-field strengths can be increased by factors of 1.7 and 2.5 for the dipole and octopole extrapolation respectively if we assume that the NS radius is around 2.5 $r_{\rm S}$ instead of 3 $r_{\rm S}$.

The magnetic field strength can be roughly checked using an equipartition estimate for which $B^2/8\pi = nkT$. For example, Sco X-1's TL density $\rho = 2.7 \times 10^{-5}$ g cm^{-3} is obtained using the best-fit optical depth $\tau = 11.3$. The TL size, $L = 10^6$ cm, is obtained using the observed Keplerian frequency and best-fit temperature $kT = 2.9$ keV. Thus, we obtain a magnetic field, $B = 1.3 \times 10^6$ G, at radius $r = 2.3 \times 10^6$ cm.

5. Summary and Conclusions

Since the discovery of neuron stars in the 1960s magnetic fields have been estimated almost exclusively from *rotational* effects, i.e. from periodic pulses with their periods p and derivatives $\dot{p}$ [e.g. Bhattacharya & Srinivasan (1995)]. The new idea here is that, when the field configuration is nearly axis-symmetric, there may arise circumstances in which the field is estimated from radial oscillations (QPOs) of plasma interacting with magnetosphere.

The model preconditions are: (1) the MA QPO must be detected observationally, (2) a Keplerian QPO frequency must be observed and (3) the temperature and the optical depth of the QPO emission region must be obtained from the photon spectra. From our model we derived tightly constrained dipole magnetic fields of the TL for the bright LMXBs 4U 1728-34, GX 340+0, and Sco X-1. The observed low frequency correlation of the kHz QPOs strongly confine the theoretical dependence of ν_{MA} on $\nu_{\rm K}$, which is mostly determined by the magnetic frequency dependence on $\nu_{\rm K}$ (see dashed lines in Figure 1) and very sensitive to the pole order (dipole, quadrupole, *etc.*). The correction to acoustic oscillations [the first term in formulas (17-18)] only slightly changes the theoretical shape of the low frequencies of $\nu_{\rm K}$.

We re-fit the stellar magnetic dipole moment in each source to derive a range of values from $(0.5 - 1) \times 10^{25}$ G cm^3. The field estimate arrived at this manner is close to that may be evolutionary end points of LMXBs, namely milisecond pulsars, which have derived values (from $\sqrt{p\dot{p}}$) of $\sim 10^8$ G (Bhattacharya 1995).

We wish to thank Sergey Kuznetsov, Eric Ford, Peter Jonker and Michael van der Klis for the Sco X-1, 4U 1728-34 and GX 340+0 data files. We are truly grateful to the referee, J. Imamura for his thorough analysis of the presented manuscript.

REFERENCES

Alfvén, H. 1942, Nature, 150, 405

Bhattacharya, D. 1995, in X-ray Binaries, eds. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel, (Cambridge: U. Press), 233

Bhattacharya, D., & Srinivasan, G. 1995, in X-ray Binaries, eds. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel, (Cambridge: U. Press), 495

Ford, E., & van der Klis, M. 1998, ApJ, 506, L39

Lang, K. R., 1998, Astrophysical Formulae, (Berlin: Springer)

Sunyaev, R. & Titarchuk, L., 1980, A & A, 86, 121

Titarchuk, L., Bradshaw, C. F., Geldzahler, B. J., & Fomalont, E. B. 2001, ApJ, 555, L45 (TBGF)

Titarchuk, L., & Osherovich, V. 2001, ApJ, 555, L55

Titarchuk, L. & Osherovich, V. 1999, ApJ, 518, L95 (TO99)

Titarchuk, L., Osherovich, V., & Kuznetsov, S., 1999, ApJ, 525, L129

Titarchuk, L., Lapidus, I.,& Muslimov, A. 1998, ApJ, 499, 315 (TLM98)

Vaughan, B. et al. 1994, ApJ, 435, 362

Wood, K.S., Titarchuk, L.G., Ray, P.S., Wolff, M.T., Lovellette, M.N., Bandyopadhyay, R.M. 2001, ApJ, in press

Wood, K.S., et al. 1991, ApJ, 379, 295

This preprint was prepared with the AAS LaTeX macros v5.0.

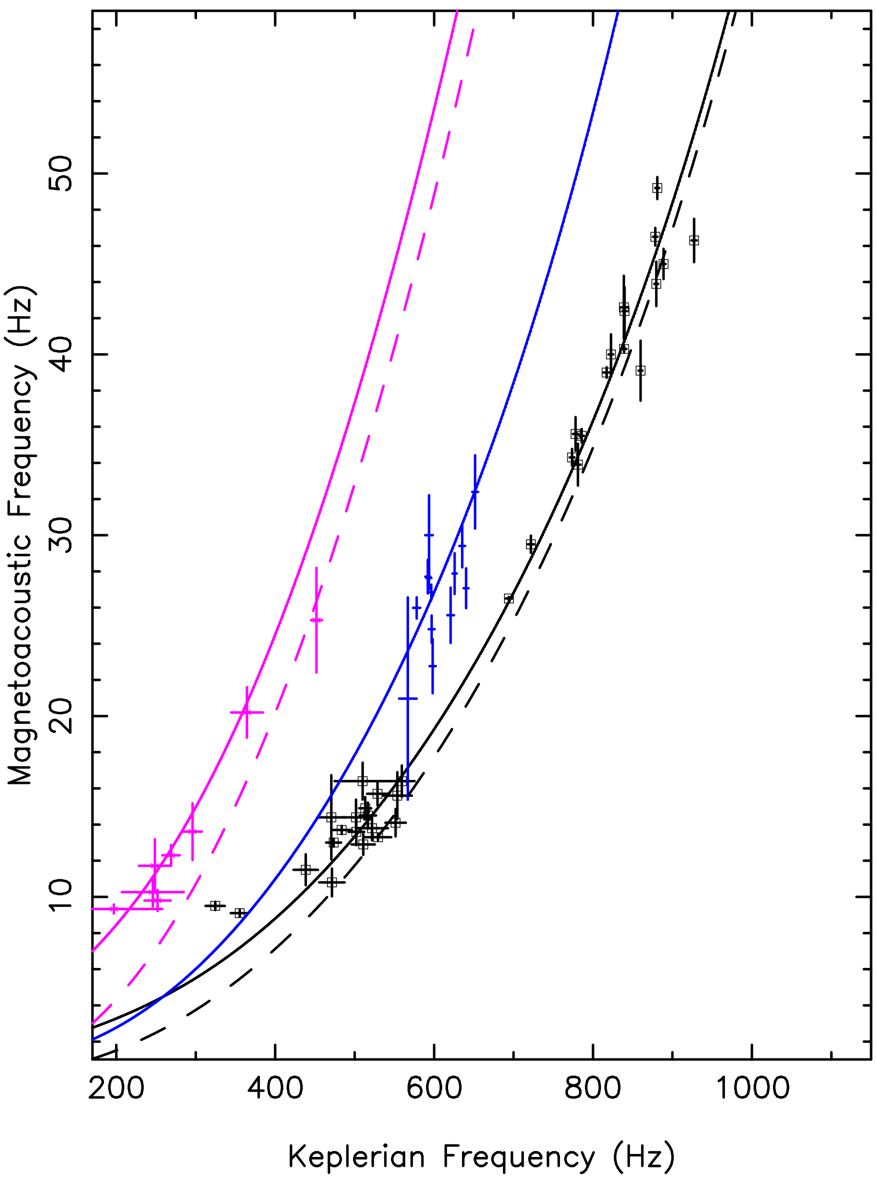

Fig. 1.— MA frequency vs Keplerian frequency for 4U 1728-34 (black squares represent QPO frequencies from Ford & van der Klis 1998, blue, from TOK and magenta, from Jonker et al. 2000). 1-σ errors bars indicate statistical fitting errors ($\Delta\chi^2 = 1$). The value for Q ($= \nu$/FWHM) approaches 1.0 for viscous QPOs corresponding to Keplerian frequencies of 320-360 Hz (Ford & van der Klis 1998), giving formal frequency errors significantly larger than the plotted error bars. The solid lines are the theoretical best-fits for the MA frequencies. The dashed lines are for the magnetic frequencies only (see text).